

# Beginning and Intermediate Algebra 

Margaret L. Lial<br>American River College

John Hornsby<br>University of New Orleans

Terry McGinnis

Vice President, Courseware Portfolio Management: Chris Hoag
Director, Courseware Portfolio Management: Michael Hirsch
Courseware Portfolio Manager: Karen Montgomery
Courseware Portfolio Assistant: Kayla Shearns
Managing Producer: Scott Disanno
Content Producer: Lauren Morse
Producers: Stacey Miller and Noelle Saligumba
Managing Producer: Vicki Dreyfus
Associate Content Producer, TestGen: Rajinder Singh
Content Managers, MathXL: Eric Gregg and Dominick Franck
Manager, Courseware QA: Mary Durnwald
Senior Product Marketing Manager: Alicia Frankel
Product Marketing Assistant: Brooke Imbornone
Senior Author Support/Technology Specialist: Joe Vetere
Full Service Vendor, Cover Design, Composition: Pearson CSC
Full Service Project Management: Pearson CSC (Carol Merrigan)
Cover Image: Borchee/E+/Getty Images
Copyright © 2020, 2016, 2012 by Pearson Education, Inc. 221 River Street, Hoboken, NJ 07030. All Rights Reserved. Printed in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise. For information regarding permissions, request forms, and the appropriate contacts within the Pearson Education Global Rights \& Permissions department, please visit www.pearsoned.com/permissions/.

Acknowledgments of third-party content appear on page C-1, which constitutes an extension of this copyright page.

PEARSON, ALWAYS LEARNING, MyLab ${ }^{\text {TM }}$ Math, MathXL, and TestGen are exclusive trademarks in the U.S. and/or other countries owned by Pearson Education, Inc. or its affiliates.

Unless otherwise indicated herein, any third-party trademarks that may appear in this work are the property of their respective owners and any references to third-party trademarks, logos or other trade dress are for demonstrative or descriptive purposes only. Such references are not intended to imply any sponsorship, endorsement, authorization, or promotion of Pearson's products by the owners of such marks, or any relationship between the owner and Pearson Education, Inc. or its affiliates, authors, licensees or distributors.

## Library of Congress Cataloging-in-Publication Data

Names: Lial, Margaret L., author. I Hornsby, John, 1949- author. I McGinnis, Terry, author.
Title: Beginning and intermediate algebra / Margaret L. Lial (American River College), John Hornsby (University of New Orleans), Terry McGinnis.
Description: 7th edition. I Boston : Pearson, [2020] I Includes index.
Identifiers: LCCN 2019000104 I ISBN 9780134895994 (student edition) I ISBN 0134895991 (student edition)
Subjects: LCSH: Algebra--Textbooks.
Classification: LCC QA152.3 .L52 2020 I DDC 512.9--dc23
LC record available at https://lcen.loc.gov/2019000104

## CONTENTS

## Preface viii

## Study Skills S-1

Study Skill 1 Using Your Math Text S-1
STUDY SKILL 2 Reading Your Math Text S-2
STUDY SKILL 3 Taking Lecture Notes S-3
STUDY SKILL 4 Completing Your Homework S-4
STUDY SKILL 5 Using Study Cards S-5

STUDY SKILL 6 Managing Your Time S-6
STUDY SKILL 7 Reviewing a Chapter S-7
STUDY SKILL 8 Taking Math Tests S-8
STUDY SKILL 9 Analyzing Your Test Results S-9
STUDY SKILL 10 Preparing for Your Math Final Exam S-10

## R Prealgebra Review 1

R. 1 Fractions 1

## 1 The Real Number System 27

R. 2 Decimals and Percents 16
1.1 Exponents, Order of Operations, and Inequality ..... 28
1.2 Variables, Expressions, and Equations ..... 36
1.3 Real Numbers and the Number Line ..... 42
1.4 Adding and Subtracting Real Numbers ..... 51
1.5 Multiplying and Dividing Real Numbers ..... 65
SUMMARY EXERCISES Performing Operations with Real Numbers ..... 77
1.6 Properties of Real Numbers ..... 78
1.7 Simplifying Expressions ..... 88
Chapter 1 Summary ..... 94
Chapter 1 Review Exercises ..... 97
Chapter 1 Mixed Review Exercises ..... 100
Chapter 1 Test ..... 100
Chapters R and 1 Cumulative Review
Exercises ..... 102
2 Linear Equations and Inequalities in One Variable ..... 103
2.1 The Addition Property of Equality ..... 104
2.2 The Multiplication Property of Equality ..... 112
2.3 Solving Linear Equations Using Both Properties of Equality ..... 117
2.4 Clearing Fractions and Decimals When Solving Linear Equations ..... 125
SUMMARY EXERCISES Applying Methods for Solving Linear Equations ..... 131
2.5 Applications of Linear Equations ..... 132
2.6 Formulas and Additional Applications from Geometry ..... 146
2.7 Ratio, Proportion, and Percent ..... 157
2.8 Further Applications of Linear Equations ..... 169
2.9 Solving Linear Inequalities ..... 182
Chapter 2 Summary ..... 196
Chapter 2 Review Exercises ..... 200
Chapter 2 Mixed Review Exercises ..... 203
Chapter 2 Test ..... 204
Chapters R-2 Cumulative ReviewExercises 205

## 3 Linear Equations in Two Variables 207

3.1 Linear Equations and Rectangular Coordinates ..... 208
3.2 Graphing Linear Equations in Two Variables ..... 219
3.3 The Slope of a Line ..... 231
3.4 Slope-Intercept Form of a Linear Equation ..... 245
3.5 Point-Slope Form of a Linear Equation and Modeling ..... 253
SUMMARY EXERCISES Applying Graphing and Equation-Writing Techniques for Lines 261
Chapter 3 Summary ..... 262
Chapter 3 Review Exercises ..... 265
Chapter 3 Mixed Review Exercises ..... 267
Chapter 3 Test ..... 268
Chapters R-3 Cumulative Review
Exercises ..... 269
4 Exponents and Polynomials ..... 271
4.1 The Product Rule and Power Rules for Exponents ..... 272
4.2 Integer Exponents and the Quotient Rule ..... 280
SUMMARY EXERCISES Applying the Rules for Exponents ..... 290
4.3 Scientific Notation ..... 291
4.4 Adding, Subtracting, and Graphing Polynomials ..... 299
4.5 Multiplying Polynomials ..... 310
4.6 Special Products ..... 317
4.7 Dividing Polynomials ..... 323
Chapter 4 Summary ..... 332
Chapter 4 Review Exercises ..... 335
Chapter 4 Mixed Review Exercises ..... 338
Chapter 4 Test ..... 338
Chapters R-4 Cumulative ReviewExercises 340
5 Factoring and Applications ..... 343
5.1 Greatest Common Factors; Factoring by Grouping ..... 344
5.2 Factoring Trinomials ..... 353
5.3 More on Factoring Trinomials ..... 360
5.4 Special Factoring Techniques ..... 369
SUMMARY EXERCISES Recognizing and Applying Factoring Strategies ..... 379
5.5 Solving Quadratic Equations Using the Zero-Factor Property ..... 382

## 6 Rational Expressions and Applications 411

6.1 The Fundamental Property of Rational
Expressions 412

6.2 Multiplying and Dividing Rational
Expressions ..... 422
6.3 Least Common Denominators ..... 429
6.4 Adding and Subtracting Rational Expressions ..... 436
6.5 Complex Fractions ..... 444
6.6 Solving Equations with Rational Expressions ..... 454
SUMMARY EXERCISES Simplifying Rational Expressionsvs. Solving RationalEquations 466
6.7 Applications of Rational Expressions ..... 468
Chapter 6 Summary ..... 478
Chapter 6 Review Exercises ..... 483
Chapter 6 Mixed Review Exercises ..... 485
Chapter 6 Test ..... 486
Chapters R-6 Cumulative Review
Exercises ..... 487
7 Linear Equations, Graphs, and Systems ..... 489
7.1 Review of Graphs and Slopes of Lines ..... 490
7.2 Review of Equations of Lines; Linear
Models
Models ..... 508 ..... 508
7.3 Solving Systems of Linear Equations by Graphing ..... 522
7.4 Solving Systems of Linear Equations by Solving Syste
Substitution ..... 531 ..... 31
7.5 Solving Systems of Linear Equations by
Elimination
Elimination ..... 539 ..... 539
SUMMARY EXERCISES Applying Techniques for Solving Systems of Linear Equations 546 SUMMARY EXERCISES Applying T
Systems o
Equations ..... 546
7.6 Systems of Linear Equations in Three Variables 548 ..... 548
7.7 Applications of Systems of Linear Equations ..... 557
Chapter 7 Summary ..... 573
Chapter 7 Review Exercises ..... 578
Chapter 7 Mixed Review Exercises ..... 582
Chapter 7 Test ..... 583

Chapters R-7 Cumulative Review

Chapters R-7 Cumulative Review
Exercises
Exercises ..... 584 ..... 584

## 9 Relations and Functions 635

9.1 Introduction to Relations and Functions 636
9.2 Function Notation and Linear Functions 647
9.3 Polynomial Functions, Graphs, Operations, and Composition 656
9.4 Variation ..... 669
Chapter 9 Summary ..... 679
Chapter 9 Review Exercises ..... 681
Chapter 9 Mixed Review Exercises ..... 683
Chapter 9 Test ..... 684
Chapters R-9 Cumulative Review Exercises ..... 685
10 Roots, Radicals, and Root Functions ..... 687
10.1 Radical Expressions and Graphs ..... 688
10.2 Rational Exponents ..... 700
10.3 Simplifying Radicals, the Distance Formula, and Circles ..... 709
10.4 Adding and Subtracting Radical Expressions ..... 723
10.5 Multiplying and Dividing Radical Expressions ..... 728
SUMMARY EXERCISES Performing Operations with
Radicals and Rational
Exponents ..... 738
10.6 Solving Equations with Radicals ..... 739
10.7 Complex Numbers ..... 746
Chapter 10 Summary ..... 754
Chapter 10 Review Exercises ..... 759
Chapter 10 Mixed Review Exercises ..... 762
Chapter 10 Test ..... 763
Chapters R-10 Cumulative Review Exercises ..... 764
11 Quadratic Equations, Inequalities, and Functions ..... 767
11.1 Solving Quadratic Equations by the Square Root Property ..... 768
11.2 Solving Quadratic Equations by Completing the Square ..... 774
11.3 Solving Quadratic Equations by the Quadratic Formula ..... 782
11.4 Equations That Lead to Quadratic Methods ..... 789
SUMMARY EXERCISES Applying Methods for Solving Quadratic Equations ..... 800
11.5 Formulas and Further Applications ..... 801
11.6 Graphs of Quadratic Functions ..... 810
11.7 More about Parabolas and Their Applications ..... 819
11.8 Polynomial and Rational Inequalities ..... 830
Chapter 11 Summary ..... 839
Chapter 11 Review Exercises ..... 843
Chapter 11 Mixed Review Exercises ..... 846
Chapter 11 Test ..... 847
Chapters R-11 Cumulative Review Exercises ..... 849
12 Inverse, Exponential, and Logarithmic Functions ..... 851
12.1 Inverse Functions ..... 852
12.2 Exponential Functions ..... 861
12.3 Logarithmic Functions ..... 870
12.4 Properties of Logarithms ..... 879
12.5 Common and Natural Logarithms ..... 886
12.6 Exponential and Logarithmic Equations; Further Applications ..... 894
13 Nonlinear Functions, Conic Sections, and Nonlinear Systems ..... 917
13.1 Additional Graphs of Functions ..... 918
13.2 Circles Revisited and Ellipses ..... 924
13.3 Hyperbolas and Functions Defined by Radicals ..... 934
13.4 Nonlinear Systems of Equations ..... 942
13.5 Second-Degree Inequalities and Systems of Inequalities ..... 948
Chapter 13 Summary ..... 955
Chapter 13 Review Exercises ..... 958
Chapter 13 Mixed Review Exercises ..... 960
Chapter 13 Test ..... 961
Chapters R-13 Cumulative Review
Chapter 12 Summary ..... 903
Chapter 12 Review Exercises ..... 907
Chapter 12 Mixed Review Exercises ..... 910
Chapter 12 Test ..... 912
Chapters R-12 Cumulative Review Exercises ..... 913
Exercises ..... 962
14 Further Topics in Algebra 965
14.1 Sequences and Series ..... 966
14.2 Arithmetic Sequences ..... 972
14.3 Geometric Sequences ..... 980
14.4 The Binomial Theorem ..... 990
Chapter 14 Summary ..... 996

## WELCOME TO THE 7TH EDITION

The first edition of Marge Lial's Beginning and Intermediate Algebra was published in 1996, and now we are pleased to present the 7th edition-with the same successful, well-rounded framework that was established 24 years ago and updated to meet the needs of today's students and professors. The names Lial and Miller, two faculty members from American River College in Sacramento, California, have become synonymous with excellence in Developmental Mathematics, Precalculus, Finite Mathematics, and Applications-Based Calculus.

With Chuck Miller's passing, Marge Lial was joined by a team of carefully selected coauthors who partnered with her. John Hornsby (University of New Orleans) joined Marge in this capacity in 1992, and in 1999, Terry McGinnis became part of this developmental author team. Since Marge's passing in 2012, John and Terry have dedicated themselves to carrying on the Lial/Miller legacy.

In the preface to the first edition of Intermediate Algebra, Marge Lial wrote
". . . the strongest theme . . . is a combination of readability and suitability for the book's intended audience: students who are not completely selfconfident in mathematics as they come to the course, but who must be self-confident and proficient . . . by the end of the course."
Today's Lial author team upholds these same standards. With the publication of the 7th edition of Beginning and Intermediate Algebra, we proudly present a complete course program for students who need developmental algebra. Revisions to the core text, working in concert with such innovations in the MyLab Math course as Skill Builder and Learning Catalytics, combine to provide superior learning opportunities appropriate for all types of courses (traditional, hybrid, online).

We hope you enjoy using it as much as we have enjoyed writing it. We welcome any feedback that you have as you review and use this text.

## WHAT'S NEW IN THIS EDITION?

We are pleased to offer the following new features and resources in the text and MyLab.

IMPROVED STUDY SKILLS These special activities are now grouped together at the front of the text, prior to Chapter R. Study Skills Reminders that refer students to specific Study Skills are found liberally throughout the text. Many Study Skills
now include a Now Try This section to help students implement the specific skill.

REVISED EXPOSITION With each edition of the text, we continue to polish and improve discussions and presentations of topics to increase readability and student understanding. This edition is no exception.

NEW FIGURES AND DIAGRAMS For visual learners, we have included more than 50 new mathematical figures, graphs, and diagrams, including several new "hand drawn" style graphs. These are meant to suggest what a student who is graphing with paper and pencil should obtain. We use this style when introducing a particular type of graph for the first time.

ENHANCED USE OF PEDAGOGICAL COLOR We have thoroughly reviewed the use of pedagogical color in discussions and examples and have increased its use whenever doing so would enhance concept development, emphasize important steps, or highlight key procedures.

INCREASED Concept Check AND what went wrong? EXERCISES The number of Concept Check exercises, which facilitate students' mathematical thinking and conceptual understanding, and which begin each exercise set, has been increased. We have also more than doubled the number of WHAT WENT WRONG? exercises that highlight common student errors.

INCREASED RELATING CONCEPTS EXERCISES We have doubled the number of these flexible groups of exercises, which are located at the end of many exercise sets. These sets of problems were specifically written to help students tie concepts together, compare and contrast ideas, identify and describe patterns, and extend concepts to new situations. They may be used by individual students or by pairs or small groups working collaboratively. All answers to these exercises appear in the student answer section.

ENHANCED MYLAB MATH RESOURCES MyLab exercise coverage in the revision has been expanded, and video coverage has also been expanded and updated to a modern format for today's students. WHAT WENT WRONG? problems and all
RELATING CONCEPTS exercise sets (both even- and oddnumbered problems) are now assignable in MyLab Math.

SKILL BUILDER These exercises offer just-in-time additional adaptive practice in MyLab Math. The adaptive engine tracks student performance and delivers, to each individual, questions that adapt to his or her level of understanding. This new feature enables instructors to assign fewer questions for
homework, allowing students to complete as many or as few questions as they need.

LEARNING CATALYTICS This new student response tool uses students' own devices to engage them in the learning process. Problems that draw on prerequisite skills are included at the beginning of each section to gauge student readiness for the section. Accessible through MyLab Math and customizable to instructors' specific needs, these problems can be used to generate class discussion, promote peer-to-peer learning, and provide real-time feedback to instructors. More information can be found via the Learning Catalytics link in MyLab Math. Specific exercises notated in the text can be found by searching LialBegIntAlg\# where the \# is the chapter number.

## CONTENT CHANGES

Specific content changes include the following:

- Exercise sets have been scrutinized and updated with a renewed focus on conceptual understanding and skill development. Even and odd pairing of the exercises, an important feature of the text, has been carefully reviewed.
- Real-world data in all examples and exercises and in their accompanying graphs has been updated.
- An increased emphasis on fractions, decimals, and percents appears throughout the text. We have expanded Chapter $\mathbf{R}$ to include new figures and revised explanations and examples on converting among fractions, decimals, and percents. And we have included an all-new set of Cumulative Review Exercises, many of which focus on fractions, decimals, and percents, at the end of Chapter 1. Sets of Cumulative Review Exercises in subsequent chapters now begin with new exercises that review skills related to these topics.
- A new Section 2.4 provides expanded coverage of linear equations in one variable with fractional and decimal coefficients. Two new examples have been included, and the number of exercises has been doubled.
- Solution sets of linear inequalities in Section 2.9 are now graphed first, before they are written using interval notation.
- Expanded Mid-Chapter Summary Exercises in Chapter 2 continue our emphasis on the difference between simplifying an expression and solving an equation. New examples in the Summary Exercises in Chapters 5 and 7 illustrate and distinguish between solution methods.
- Chapters 13 and 14 on Nonlinear Functions, Conic Sections, Nonlinear Systems, and Further Topics in Algebra, previously available online in MyLab Math, are now included in the text. The material has been fully revised and updated.
- Presentations of the following topics have been enhanced and expanded, often including new examples and exercises.

Order of operations involving absolute value expressions (Section 1.5)
Solving linear equations in one variable (Sections 2.1, 2.2)
Solving problems involving proportions and percent (Section 2.7)
Writing an equation of a line from a graph (Section 3.4)
Adding, subtracting, and dividing polynomials (Sections 4.4 and 4.7)
Finding reciprocals of rational expressions (Section 6.2)
Geometric interpretation of slope as rise/run (Section 7.1)
Solving systems of equations using the elimination method (Section 7.5)
Solving systems of linear equations in three variables (Section 7.6)
Identifying functions and domains from equations (Section 9.1)
Graphing polynomial functions (Section 9.3)
Concepts and relationships among real numbers, nonreal complex numbers, and imaginary numbers; simplifying powers of $i$ (Section 10.7)
Solving quadratic equations using the quadratic formula (Section 11.3)
Solving exponential and logarithmic equations (Sections 12.2, 12.3)

## LIAL DEVELOPMENTAL HALLMARK FEATURES

We have enhanced the following popular features, each of which is designed to increase ease of use by students and/ or instructors.

- Emphasis on Problem-Solving We introduce our sixstep problem-solving method in Chapter 2 and integrate it throughout the text. The six steps, Read, Assign a Variable, Write an Equation, Solve, State the Answer, and Check, are emphasized in boldface type and repeated in examples and exercises to reinforce the problem-solving process for students. We also provide students with PROBLEM-SOLVING HINT boxes that feature helpful problem-solving tips and strategies.
- Helpful Learning Objectives We begin each section with clearly stated, numbered objectives, and the included material is directly keyed to these objectives so that students and instructors know exactly what is covered in each section.
- Cautions and Notes One of the most popular features of previous editions is our inclusion of information marked (I) CAUTION and NOTE to warn students about common errors and to emphasize important ideas throughout the exposition. The updated text design makes them easy to spot.
- Comprehensive Examples The new edition features a multitude of step-by-step, worked-out examples that include pedagogical color, helpful side comments, and special pointers. We give special attention to checking example solutions-more checks, designated using a special CHECK tag and $\checkmark$, are included than in past editions.
- More Pointers There are more pointers in examples and discussions throughout this edition of the text. They provide students with important on-the-spot reminders, as well as warnings about common pitfalls.
- Numerous Now Try Problems These margin exercises, with answers immediately available at the bottom of the page, have been carefully written to correspond to every example in the text. This key feature allows students to immediately practice the material in preparation for the exercise sets.
- Updated Figures, Photos, and Hand-Drawn Graphs Today's students are more visually oriented than ever. As a result, we provide detailed mathematical figures, diagrams, tables, and graphs, including a "hand-drawn" style of graphs, whenever possible. We have incorporated depictions of well-known mathematicians, as well as appealing photos to accompany applications in examples and exercises.
- Relevant Real-Life Applications We include many new or updated applications from fields such as business, pop culture, sports, technology, and the health sciences that show the relevance of algebra to daily life.
- Extensive and Varied Exercise Sets The text contains a wealth of exercises to provide students with opportunities to practice, apply, connect, review, and extend the skills they are learning. Numerous illustrations, tables, graphs, and photos help students visualize the problems they are solving. Problem types include skill building and writing exercises, as well as applications, matching, true/false, multiple-choice, and fill-in-theblank problems. Special types of exercises include Concept Check, WHAT WENT WRONG? , Extending Skills, and RELATING CONCEPTS .
- Special Summary Exercises We include a set of these popular in-chapter exercises in every chapter. They provide students with the all-important mixed review problems they need to master topics and often include summaries of solution methods and/or additional examples.
- Extensive Review Opportunities We conclude each chapter with the following review components:

A Chapter Summary that features a helpful list of Key Terms organized by section, New Symbols, a Test Your Word Power vocabulary quiz (with answers immediately following), and a Quick Review of each section's main concepts, complete with additional examples.

A comprehensive set of Chapter Review Exercises, keyed to individual sections for easy student reference.

A set of Mixed Review Exercises that helps students further synthesize concepts and skills.

A Chapter Test that students can take under test conditions to see how well they have mastered the chapter material.
A set of Cumulative Review Exercises for ongoing review that covers material going back to Chapter R.

- Comprehensive Glossary The online Glossary includes key terms and definitions (with section references) from throughout the text.


## ACKNOWLEDGMENTS

The comments, criticisms, and suggestions of users, nonusers, instructors, and students have positively shaped this text over the years, and we are most grateful for the many responses we have received. The feedback gathered for this edition was particularly helpful.

We especially wish to thank the following individuals who provided invaluable suggestions.

Barbara Aaker, Community College of Denver<br>Kim Bennekin, Georgia Perimeter College<br>Dixie Blackinton, Weber State University<br>Eun Cha, College of Southern Nevada, Charleston<br>Callie Daniels, St. Charles Community College<br>Cheryl Davids, Central Carolina Technical College Robert Diaz, Fullerton College<br>Chris Diorietes, Fayetteville Technical Community College<br>Sylvia Dreyfus, Meridian Community College<br>Sabine Eggleston, Edison State College<br>LaTonya Ellis, Bishop State Community College<br>Beverly Hall, Fayetteville Technical Community College<br>Aaron Harris, College of Southern Nevada, Charleston<br>Loretta Hart, NHTI, Concord's Community College<br>Sandee House, Georgia Perimeter College<br>Joe Howe, St. Charles Community College<br>Lynette King, Gadsden State Community College<br>Linda Kodama, Windward Community College<br>Carlea McAvoy, South Puget Sound Community College<br>James Metz, Kapi’olani Community College<br>Jean Millen, Georgia Perimeter College<br>Molly Misko, Gadsden State Community College<br>Charles Patterson, Louisiana Tech<br>Jane Roads, Moberly Area Community College<br>Melanie Smith, Bishop State Community College<br>Erik Stubsten, Chattanooga State Technical Community College<br>Tong Wagner, Greenville Technical College<br>Rick Woodmansee, Sacramento City College<br>Sessia Wyche, University of Texas at Brownsville

Over the years, we have come to rely on an extensive team of experienced professionals. Our sincere thanks go to these dedicated individuals at Pearson who worked long and hard to make this revision a success.

We would like to thank Michael Hirsch, Matthew Summers, Karen Montgomery, Alicia Frankel, Lauren Morse, Vicki Dreyfus, Stacey Miller, Noelle Saligumba, Eric Gregg, and all of the Pearson math team for helping with the revision of the text.

We are especially pleased to welcome Callie Daniels, who has taught from our texts for many years, to our team. Her assistance has been invaluable. She thoroughly reviewed all chapters and helped extensively with manuscript preparation.

We are grateful to Carol Merrigan for her excellent production work. We appreciate her positive attitude, responsiveness, and expert skills. We would also like
to thank Pearson CSC for their production work; Emily Keaton for her detailed help in updating real data applications; Connie Day for supplying her copyediting expertise; Pearson CSC for their photo research; and Lucie Haskins for producing another accurate, useful index. Paul Lorczak and Hal Whipple did a thorough, timely job accuracychecking the page proofs and answers, and Sarah Sponholz checked the index.

We particularly thank the many students and instructors who have used this text over the years. You are the reason we do what we do. It is our hope that we have positively impacted your mathematics journey. We would welcome any comments or suggestions you might have via email to math@pearson.com.

John Hornsby Terry McGinnis

# To BK and Vangie 

E.J.H.

To Andrew and Tyler

## Mom

## Resources for Success

## Get the Most Out of MyLab Math for Beginning and Intermediate Algebra, Seventh Edition by Lial, Hornsby, McGinnis

The Lial team has helped thousands of students learn algebra with an approachable, teacherly writing style and balance of skill and concept development. With this revision, the series retains the hallmarks that have helped students succeed in math, and includes new and updated digital tools in the MyLab Math course.
Take advantage of the following resources to get the most out of your MyLab Math course.

## Get Students Prepared with Integrated Review

Every student enters class with different levels of preparedness and prerequisite knowledge. To ensure students are caught up on prior skills, every Lial MyLab course now includes Integrated Review.


## Personalize Learning

New! Skill Builder exercises offer
just-in-time additional adaptive practice. The adaptive engine tracks student performance and delivers questions to each individual that adapt to his or her level of understanding. This new feature allows instructors to assign fewer questions for homework, allowing students to complete as many or as few questions as they need.

New! Integrated Review provides embedded and personalized review of prerequisite topics within relevant chapters. Students can check their prerequisite skills, and receive personalized practice on the topics they need to focus on, with study aids like worksheets and videos also available to help.

Integrated Review assignments are premade and available to assign in the Assignment Manager.


## Resources for Success

## Support Students Whenever, Wherever

 Updated! The complete video program for the Lial series includes:- Full Section Lecture Videos
- Solution clips for select exercises
- Chapter Test Prep videos
- Short Quick Review videos that recap each section

Full Section Lecture Videos are also available as shorter, objective-level videos. No matter your students' needs-if they missed class, need help solving a problem, or want a short summary of a section's concepts-they can get support whenever they need it, wherever they need it. Much of the video series has been updated in a modern presentation format.


## Foster a Growth Mindset

New! A Mindset module is available in the course, with mindset-focused videos and exercises that encourage students to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as a learning opportunity.

## Get Students Engaged

New! Learning Catalytics Learning Catalytics is an interactive student response tool that uses students' smartphones, tablets, or laptops to engage them in more sophisticated tasks and thinking.

In addition to a library of developmental math questions, Learning Catalytics questions created specifically for this text are pre-built to make it easy for instructors to begin using this tool! These questions, which cover prerequisite skills before each section, are noted in the margin of the Annotated Instructor's Edition, and can be found in Learning Catalytics by searching for "LialBegIntAlg\#", where \# is the chapter number.


## Resources for Success

## Instructor Resources

## Annotated Instructor's Edition

Contains all the content found in the student edition, plus answers to even and odd exercises on the same text page, and Teaching Tips and Classroom Examples throughout the text placed at key points.

The resources below are available through Pearson's Instructor Resource Center, or from MyLab Math.

## Instructor's Resource Manual with Tests

Includes mini-lectures for each text section, several forms of tests per chapter-two diagnostic pretests, four free-response and two multiple-choice test forms per chapter, and two final exams.

## Instructor's Solutions Manual

Contains detailed, worked-out solutions to all exercises in the text.

## TestGen ${ }^{\circledR}$

Enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions.

## PowerPoint Lecture Slides

Available for download only, these slides present key concepts and definitions from the text. Accessible versions of the PowerPoint slides are also available for students who are visionimpaired.

## Student Resources

## Guided Notebook

This Guided Notebook helps students keep their work organized as they work through their course. The notebook includes:

- Guided Examples that are worked out for students, plus corresponding Now Try This exercises for each text objective.
- Extra practice exercises for every section of the text, with ample space for students to show their work.
- Learning objectives and key vocabulary terms for every text section, along with vocabulary practice problems.


## Student Solutions Manual

Provides completely worked-out solutions to the odd-numbered section exercises and to all exercises in the Now Trys, Relating Concepts, Chapter Reviews, Mixed Reviews, Chapter Tests, and Cumulative Reviews. Available at no additional charge in the MyLab Math course.

## Using Your Math Text

Your text is a valuable resource. You will learn more if you make full use of the features it offers.

## Now TRY THIS

## General Features of This Text

Locate each feature, and complete any blanks.

- Table of Contents This is located at the front of the text. Find it and mark the chapters and sections you will cover, as noted on your course syllabus.
- Answer Section This is located at the back of the text. Tab this section so you can easily refer to it when doing homework or reviewing for tests.
- List of Formulas This helpful list of geometric formulas, along with review information on triangles and angles, is found at the back of the text.
The formula for the volume of a cube is $\qquad$ -


## Specific Features of This Text

Look through Chapter 1 or 2 and give the number of a page that includes an example of each of the following specific features.

- Objectives The objectives are listed at the beginning of each section and again within the section as the corresponding material is presented. Once you finish a section, ask yourself if you have accomplished them.
 See page $\qquad$ —.
- Vocabulary List Important vocabulary is listed at the beginning of each section. You should be able to define these terms when you finish a section. See page $\qquad$ -.
- Now Try Exercises These margin exercises allow you to immediately practice the material covered in the examples and prepare you for the exercises. Check your results using the answers at the bottom of the page. See page $\qquad$ —.
- Pointers These small, shaded balloons provide on-the-spot warnings and reminders, point out key steps, and give other helpful tips. See page $\qquad$ -.
- Cautions These provide warnings about common errors that students often make or trouble spots to avoid. See page $\qquad$ —.
- Notes These provide additional explanations or emphasize other important ideas. See page $\qquad$ —.
- Problem-Solving Hints These boxes give helpful tips or strategies to use when you work applications. Look for them beginning in Chapter 2. See page $\qquad$ —.


## STUDY SKILL 2



## Reading Your Math Text

Take time to read each section and its examples before doing your homework. You will learn more and be better prepared to work the exercises your instructor assigns.

## Approaches to Reading Your Math Text

Student A learns best by listening to her teacher explain things. She "gets it" when she sees the instructor work problems. She previews the section before the lecture, so she knows generally what to expect. Student A carefully reads the section in her text AFTER she hears the classroom lecture on the topic.

Student B learns best by reading on his own. He reads the section and works through the examples before coming to class. That way, he knows what the teacher is going to talk about and what questions he wants to ask. Student B carefully reads the section in his text BEFORE he hears the classroom lecture on the topic.

Which of these reading approaches works best for you-that of Student A or Student B?

## Tips for Reading Your Math Text

- Turn off your cell phone and the TV. You will be able to concentrate more fully on what you are reading.
- Survey the material. Glance over the assigned material to get an idea of the "big picture." Look at the list of objectives to see what you will be learning.
- Read slowly. Read only one section-or even part of a section-at a sitting, with paper and pencil in hand.
- Pay special attention to important information given in colored boxes or set in boldface type. Highlight any additional information you find helpful.
- Study the examples carefully. Pay particular attention to the blue side comments and any pointer balloons.
- Do the Now Try exercises in the margin on separate paper as you go. These problems mirror the examples and prepare you for the exercise set. Check your answers with those given at the bottom of the page.
- Make study cards as you read. Make cards for new vocabulary, rules, procedures, formulas, and sample problems.
- Mark anything you don't understand. ASK QUESTIONS in class-everyone will benefit. Follow up with your instructor, as needed.


## Now TRY THIS

Think through and answer each question.

1. Which two or three reading tips given above will you try this week?
2. Did the tips you selected improve your ability to read and understand the material? Explain.

## Taking Lecture Notes

## Come to class prepared.

- Bring paper, pencils, notebook, text, completed homework, and any other materials you need.
- Arrive 10-15 minutes early if possible. Use the time before class to review your notes or study cards from the last class period.
- Select a seat carefully so that you can hear and see what is going on.


## Study the set of sample math notes given at the right.

- Include the date and the title of the day's lecture topic.
- Include definitions, written here in parentheses - don't trust your memory.
- Skip lines and write neatly to make reading easier.
- Emphasize direction words (like evaluate, simplify, or solve) with their explanations.
- Mark important concepts with stars, underlining, etc.
- Use two columns, which allows an example and its explanation to be close together.
- Use brackets and arrows to clearly show steps, related material, etc.
- Highlight any material and/or information that your instructor emphasizes. Instructors often give "clues" about material that will definitely be on an exam.


Consider using a three-ring binder to organize your notes, class handouts, and completed homework.

## Now TRY THIS

With a study partner or in a small group, compare lecture notes. Then answer each question.

1. What are you doing to show main points in your notes (such as boxing, using stars, etc.)?
2. In what ways do you set off explanations from worked problems and subpoints (such as indenting, using arrows, circling, etc.)?
3. What new ideas did you learn by examining your classmates' notes?
4. What new techniques will you try when taking notes in future lectures?

## STUDY SKILL 4



## Completing Your Homework

You are ready to do your homework AFTER you have read the corresponding text section and worked through the examples and Now Try exercises.

## Homework Tips

- Keep distractions and potential interruptions to a minimum. Turn off your cell phone and the TV. Find a quiet, comfortable place to work, away from a lot of other people, so you can concentrate on what you are doing.
- Review your class notes. Pay particular attention to anything your instructor emphasized during the lecture on this material.
- Survey the exercise set. Glance over the problems that your instructor has assigned to get a general idea of the types of exercises you will be working. Skim directions, and note any references to section examples.
- Work problems neatly. NEVER do your math homework in pen. Use pencil and write legibly, so others can read your work. Skip lines between steps. Clearly separate problems from each other.
- Show all your work. It is tempting to take shortcuts. Include ALL steps.
- Check your work frequently to make sure you are on the right track. It is hard to unlearn a mistake. For all odd-numbered problems, answers are given in the back of the text.
- If you have trouble with a problem, refer to the corresponding worked example in the section. The exercise directions will often reference specific examples to review. Pay attention to every line of the worked example to see how to get from step to step.
- If you have trouble with an even-numbered problem, work the corresponding oddnumbered problem. Check your answer in the back of the text, and apply the same steps to work the even-numbered problem.
- If you have genuinely tried to work a problem but have not been able to complete it in a reasonable amount of time, it's ok to STOP. Mark these problems. Ask for help at your school's tutor center or from fellow classmates, study partners, or your instructor.
- Do some homework problems every day. This is a good habit, even if your math class does not meet each day.


## Now TRY THIS

Think through and answer each question.

1. What is your instructor's policy regarding homework?
2. Think about your current approach to doing homework. Be honest in your assessment.
(a) What are you doing that is working well?
(b) What improvements could you make?
3. Which one or two homework tips will you try this week?
4. In the event that you need help with homework, what resources are available? When does your instructor hold office hours?

## Using Study Cards

You may have used "flash cards" in other classes. In math, "study cards" can help you remember terms and definitions, procedures, and concepts. Use study cards to

- Help you understand and learn the material;
- Quickly review when you have a few minutes;
- Review before a quiz or test.


## One of the advantages of study cards is that you learn the material while you are making them.

## Vocabulary Cards

Put the word and a page reference on the front of the card. On the back, write the definition, an example, any related words, and a sample problem (if appropriate).

## Procedure ("Steps") Cards

Write the name of the procedure on the front of the card. Then write each step in words. On the back of the card, put an example showing each step.



## Practice Problem Cards

Write a problem with direction words (like solve, simplify) on the front of the card, and work the problem on the back. Make one for each type of problem you learn.

## Now TRY THIS

Make a vocabulary card, a procedure card, and a practice problem card for material that you are learning or reviewing.


## STUDY SKILL 6

## Managing Your Time

Many college students juggle a busy schedule and multiple responsibilities, including school, work, and family demands.


## Time Management Tips

- Read the syllabus for each class. Understand class policies, such as attendance, late homework, and make-up tests. Find out how you are graded.
- Make a semester or quarter calendar. Put test dates and major due dates for all your classes on the same calendar. Try using a different color for each class.
- Make a weekly schedule. After you fill in your classes and other regular responsibilities, block off some study periods. Aim for 2 hours of study for each 1 hour in class.
- Choose a regular study time and place (such as the campus library). Routine helps.
- Keep distractions to a minimum. Get the most out of the time you have set aside for studying by limiting interruptions. Turn off your cell phone. Take a break from social media. Avoid studying in front of the TV.
- Make "to-do" lists. Number tasks in order of importance. To see your progress, cross off tasks as you complete them.
- Break big assignments into smaller chunks. Don't wait until the last minute to begin big assignments or to study for tests. Make deadlines for each smaller chunk so that you stay on schedule.
- Take breaks when studying. Do not try to study for hours at a time. Take a 10-minute break each hour or so.
- Ask for help when you need it. Talk with your instructor during office hours. Make use of the learning/tutoring center, counseling office, or other resources available at your school.


## Now TRY THIS

Work through the following, answering any questions.

1. Evaluate when and where you are currently studying. Are these places quiet and comfortable? Are you studying when you are most alert?
2. Which of the above tips will you try this week to improve your time management?
3. Create a weekly calendar that includes your class times, study times, and other family and/or work obligations.
4. Once the week is over, evaluate how these tips worked. Did you use your calendar and stick to it? What will you do differently next week?
5. Ask classmates, friends, and/or family members for tips on how they manage their time. Try any that you think might work for you.

## Reviewing a Chapter

Your text provides extensive material to help you prepare for quizzes or tests in this course. Refer to the Chapter 1 Summary as you read through the following techniques.

## Techniques for Reviewing a Chapter

- Review the Key Terms and any New Symbols. Make a study card for each. Include a definition, an example, a sketch (if appropriate), and a section or page reference.
- Take the Test Your Word Power quiz to check your understanding of new vocabulary. The answers immediately follow.
- Read the Quick Review. Pay special attention to the headings. Study the explanations and examples given for each concept. Try to think about the whole chapter.
- Reread your lecture notes. Focus on what your instructor has emphasized in class, and review that material in your text.
- Look over your homework. Pay special attention to any trouble spots.
- Work the Review Exercises. They are grouped by section. Answers are included at the back of the text.

- Pay attention to direction words, such as simplify, solve, and evaluate.
- Are your answers exact and complete? Did you include the correct labels, such as \$, $\mathrm{cm}^{2}$, ft , etc.?
- Make study cards for difficult problems.
- Work the Mixed Review Exercises. They are in random order. Check your answers in the answer section at the back of the text.
- Take the Chapter Test under test conditions.
- Time yourself.
- Use a calculator or notes only if your instructor permits them on tests.
- Take the test in one sitting.
- Show all your work.
- Check your answers in the answer section. Section references are provided.

Reviewing a chapter takes time. Avoid rushing through your review in one night. Use the suggestions over a few days or evenings to better understand and remember the material.

## Now TRY THIS

Follow these reviewing techniques to prepare for your next test. Then answer each question.

1. How much time did you spend reviewing for your test? Was it enough?
2. Which reviewing techniques worked best for you?
3. Are you investing enough time and effort to really know the material and set yourself up for success? Explain.
4. What will you do differently when reviewing for your next test?

## STUDY SKILL 8

## Taking Math Tests

## Techniques to Improve Your Test Score

Come prepared with a pencil, eraser, paper, and calculator, if allowed.

Scan the entire test, note the point values of different problems, and plan your time accordingly.
Do a "knowledge dump" when you get the test. Write important notes, such as formulas, in a corner of the test for reference.

Read directions carefully, and circle any significant words. When you finish a problem, reread the directions. Did you do what was asked?
Show all your work. Many teachers give partial credit if some steps are correct, even if the final answer is wrong. Write neatly.
Write down anything that might help solve a problem: a formula, a diagram, etc. If necessary, circle the problem and come back to it later. Do not erase anything you wrote down.

If you can't solve a problem, make a guess. Do not change it unless you find an obvious mistake.

Check that the answer to an application problem is reasonable and makes sense. Reread the problem to make sure you've answered the question.

Check for careless errors. Rework each problem without looking at your previous work. Then compare the two answers.

## Comments

Working in pencil lets you erase, keeping your work neat.

To do 20 problems in 50 minutes, allow $50 \div 20=2.5$ minutes per problem. Spend less time on easier problems.

Writing down tips and other special information that you've learned at the beginning allows you to relax as you take the test.

Pay attention to any announcements written on the board or made by your instructor. Ask if you don't understand something.
If your teacher can't read your writing, you won't get credit for it. If you need more space to work, ask to use extra paper.

If you know even a little bit about a problem, write it down. The answer may come to you as you work on it, or you may get partial credit. Don't spend too long on any one problem.

Have a good reason for changing an answer. Your first guess is usually your best bet.

Use common sense. Can the father really be seven years old? Would a month's rent be $\$ 32,140$ ? Remember to label your answer if needed: \$, years, inches, etc.

Reworking a problem from the beginning forces you to rethink it. If possible, use a different method to solve the problem.


## Now TRY THIS

Think through and answer each question.

1. What two or three tips will you try when you take your next math test?
2. How did the tips you selected work for you when you took your math test?
3. What will you do differently when taking your next math test?
4. Ask several classmates how they prepare for math tests. Did you learn any new preparation ideas?

## Analyzing Your Test Results

An exam is a learning opportunity-learn from your mistakes. After a test is returned, do the following:

- Note what you got wrong and why you had points deducted.
- Figure out how to solve the problems you missed. Check your text or notes, or ask your instructor. Rework the problems correctly.
- Keep all quizzes and tests that are returned to you. Use them to study for future tests and the final exam.


## Typical Reasons for Errors on Math Tests

1. You read the directions wrong.
2. You read the question wrong or skipped over something.
3. You made a computation error.
4. You made a careless error. (For example, you incorrectly copied a correct answer onto a separate answer sheet.)
5. Your answer was not complete.
6. You labeled your answer wrong. (For example, you labeled an answer " ft " instead of " ft ².")
7. You didn't show your work.
8. You didn't understand a concept.
9. You were unable to set up the problem (in an application).
10. You were unable to apply a procedure.

These are test-taking errors. They are easy to correct if you read carefully, show all your work, proofread, and doublecheck units and labels.

These are test preparation errors. Be sure to practice all the kinds of problems that you will see on tests.

## Now TRY THIS

Work through the following, answering any questions.

1. Use the sample charts at the right to track your test-taking progress. Refer to the tests you have taken so far in your course. For each test, check the appropriate box in the charts to indicate that you made an error in a particular category.
2. What test-taking errors did you make? Do you notice any patterns?
3. What test preparation errors did you make? Do you notice any patterns?
4. What will you do to avoid these kinds of errors on your next test?
$\nabla$ Test-Taking Errors

| Test | Read <br> directions <br> wrong | Read <br> question <br> wrong | Made <br> computation <br> error | Made <br> careless <br> error | Answer <br> not <br> complete | Answer <br> labeled <br> wrong | Didn't <br> show <br> work |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |

V Test Preparation Errors

| Test | Didn't understand <br> concept | Didn't set up problem <br> correctly | Couldn't apply <br> a procedure |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |

## STUDY SKILL 10



## Preparing for Your Math Final Exam

Your math final exam is likely to be a comprehensive exam, which means it will cover material from the entire term. One way to prepare for it now is by working a set of Cumulative Review Exercises each time your class finishes a chapter. This continual review will help you remember concepts and procedures as you progress through the course.

## Final Exam Preparation Suggestions

1. Figure out the grade you need to earn on the final exam to get the course grade you want. Check your course syllabus for grading policies, or ask your instructor if you are not sure.
2. Create a final exam week plan. Set priorities that allow you to spend extra time studying. This may mean making adjustments, in advance, in your work schedule or enlisting extra help with family responsibilities.
3. Use the following suggestions to guide your studying.

- Begin reviewing several days before the final exam. DON'T wait until the last minute.
- Know exactly which chapters and sections will be covered on the exam.
- Divide up the chapters. Decide how much you will review each day.
- Keep returned quizzes and tests. Use them to review.
- Practice all types of problems. Use the Cumulative Review Exercises at the end of each chapter in your text beginning in Chapter 1. All answers, with section references, are given in the answer section at the back of the text.
- Review or rewrite your notes to create summaries of important information.
- Make study cards for all types of problems. Carry the cards with you, and review them whenever you have a few minutes.
- Take plenty of short breaks as you study to reduce physical and mental stress. Exercising, listening to music, and enjoying a favorite activity are effective stress busters.

Finally, DON'T stay up all night the night before an exam-get a good night's sleep.

## Now TRY THIS

Think through and answer each question.

1. How many points do you need to earn on your math final exam to get the grade you want in your course?
2. What adjustments to your usual routine or schedule do you need to make for final exam week? List two or three.
3. Which of the suggestions for studying will you use as you prepare for your math final exam? List two or three.
4. Analyze your final exam results. How will you prepare differently next time?

## PREALGEBRA REVIEW

R. 1 Fractions
R. 2 Decimals and Percents

## R. 1 Fractions

## OBJECTIVES

1 Write numbers in factored form.
2 Write fractions in lowest terms.
3 Convert between improper fractions and mixed numbers.
4 Multiply and divide fractions.
5 Add and subtract fractions.
6 Solve applied problems that involve fractions.
7 Interpret data in a circle graph.

## VOCABULARY

$\square$ natural (counting) numbers
$\square$ whole numbers
$\square$ fractions
$\square$ numerator
$\square$ denominator
$\square$ proper fraction
$\square$ improper fraction
$\square$ factors
$\square$ product
$\square$ prime number
(continued)

The numbers used most often in everyday life are the natural (counting) numbers,

$$
1,2,3,4, \ldots,
$$

the whole numbers,

$$
0,1,2,3,4, \ldots
$$

The three dots, or ellipsis points,
indicate that each list of numbers continues in the same way indefinitely. and fractions, such as

$$
\frac{1}{2}, \quad \frac{2}{3}, \text { and } \frac{11}{12}
$$

The parts of a fraction are named as shown.

$$
\text { Fraction bar } \rightarrow \frac{3}{8} \leftarrow \text { Numerator }
$$

The fraction bar represents division $\left(\frac{a}{b}=a \div b\right)$.

NOTE Fractions are a way to represent parts of a whole. In a fraction, the numerator gives the number of parts being represented. The denominator gives the total number of equal parts in the whole. See FIGURE 1.


The shaded region represents
$\frac{3}{8}$ of the circle.

A fraction is classified as being either a proper fraction or an improper fraction.

| Proper fractions | $\frac{1}{5}$, |
| ---: | :--- |
| $\frac{2}{7}$, | $\frac{9}{10}, \frac{23}{25}$ |
| Numerator is less than denominator. <br> Value is less than 1. |  |
| Improper fractions | $\frac{3}{2}$, |$\frac{5}{5}, \frac{11}{7}, \frac{28}{4}$| Numerator is greater than or |
| :--- |
| equal to denominator. Value is |
| greater than or equal to 1. |

Numerator is less than denominator. Value is less than 1.

Numerator is greater than or equal to denominator. Value is greater than or equal to 1 .

## VOCABULARY (continued)

composite number
$\square$ lowest terms
$\square$ mixed number
$\square$ reciprocals
$\square$ quotient
$\square$ dividend
$\square$ divisor
$\square$ sum
$\square$ least common denominator (LCD)
$\square$ difference
$\square$ circle graph (pie chart)

## OBJECTIVE 1 Write numbers in factored form.

In the statement $3 \times 6=18$, the numbers 3 and 6 are factors of 18 . Other factors of 18 include $1,2,9$, and 18 . The result of the multiplication, 18 , is the product. We can represent the product of two numbers, such as 3 and 6 , in several ways.

$$
3 \times 6, \quad 3 \cdot 6, \quad(3)(6), \quad(3) 6, \quad 3(6) \quad \text { Products }
$$

We factor a number by writing it as the product of two or more numbers.


NOTE In algebra, a raised dot •is often used instead of the $\times$ symbol to indicate multiplication because $\times$ may be confused with the letter $x$.

A natural number greater than 1 is prime if it has only itself and 1 as factors. "Factors" are understood here to mean natural number factors.

$$
2,3,5,7,11,13,17,19,23,29,31,37 \quad \text { First dozen prime numbers }
$$

A natural number greater than 1 that is not prime is a composite number.

$$
4,6,8,9,10,12,14,15,16,18,20,21 \quad \text { First dozen composite numbers }
$$

The number 1 is considered to be neither prime nor composite.

NOW TRY
EXERCISE 1
Identify the number 60 as prime, composite, or neither. If the number is composite, write it as a product of prime factors.

NOW TRY ANSWER

1. composite; $2 \cdot 2 \cdot 3 \cdot 5$

## EXAMPLE 1 Writing Numbers in Prime Factored Form

Identify each number as prime, composite, or neither. If the number is composite, write it as a product of prime factors.
(a) 43

There are no natural numbers other than 1 and 43 itself that divide evenly into 43, so the number 43 is prime.
(b) 35

The number 35 is composite and can be written as the product of the prime factors 5 and 7.

$$
35=5 \cdot 7
$$

(c) 24

The number 24 is composite. We show a factor tree on the right, with prime factors circled.

Divide by the least prime factor of 24 , which is 2 .

$$
24=2 \cdot 12
$$



Divide 12 by 2 to find two factors of 12 .
$24=2 \cdot 2 \cdot 6$

Now factor 6 as $2 \cdot 3$.

$$
24=2 \cdot 2 \cdot 2 \cdot 3
$$



All factors are prime.

NOTE No matter which prime factor we start with when factoring, we will always obtain the same prime factorization. We verify this in Example 1(c) by starting with 3 instead of 2.


## OBJECTIVE 2 Write fractions in lowest terms.

The following properties are useful when writing a fraction in lowest terms.

## Properties of 1

Any nonzero number divided by itself is equal to 1 . Example: $\frac{3}{3}=1$
Any number multiplied by 1 remains the same. Example: $\frac{2}{5} \cdot 1=\frac{2}{5}$

A fraction is in lowest terms when the numerator and denominator have no factors in common (other than 1).

## Writing a Fraction in Lowest Terms

Step 1 Write the numerator and denominator in factored form.
Step 2 Replace each pair of factors common to the numerator and denominator with 1.

Step 3 Multiply the remaining factors in the numerator and in the denominator. (This procedure is sometimes called "simplifying the fraction.")

## EXAMPLE 2 Writing Fractions in Lowest Terms

Write each fraction in lowest terms.
(a) $\frac{10}{15}=\frac{2 \cdot 5}{3 \cdot 5}=\frac{2}{3} \cdot \frac{5}{5}=\frac{2}{3} \cdot 1=\frac{2}{3} \quad \begin{aligned} & \text { Use the first property of } 1 \\ & \text { to replace } \frac{5}{5} \text { with } 1 .\end{aligned}$
(b) $\frac{15}{45}$

By inspection, the greatest common factor of 15 and 45 is 15 .

$$
\frac{15}{45}=\frac{15}{3 \cdot 15}=\frac{1}{3 \cdot 1}=\frac{1}{3} \quad \begin{aligned}
& \begin{array}{l}
\text { Remember to witie } \\
\text { in in the numerator. }
\end{array}
\end{aligned}
$$

If the greatest common factor is not obvious, factor the numerator and denominator into prime factors.

$$
\frac{15}{45}=\frac{3 \cdot 5}{3 \cdot 3 \cdot 5}=\frac{1 \cdot 1}{3 \cdot 1 \cdot 1}=\frac{1}{3} \quad \text { The same answer results. }
$$

NOW TRY
EXERCISE 2
Write each fraction in lowest terms.
(a) $\frac{30}{42}$
(b) $\frac{10}{70}$
(c) $\frac{72}{120}$

NOW TRY
EXERCISE 3
Write $\frac{92}{5}$ as a mixed number.

NOW TRY
EXERCISE 4
Write $11 \frac{2}{3}$ as an improper fraction.
(c) $\frac{150}{200}=\frac{3 \cdot 50}{4 \cdot 50}=\frac{3}{4} \cdot 1=\frac{3}{4} \quad 50$ is the greatest common factor of 150 and 200 .

Another strategy is to choose any common factor and work in stages.

$$
\frac{150}{200}=\frac{15 \cdot 10}{20 \cdot 10}=\frac{3 \cdot 5 \cdot 10}{4 \cdot 5 \cdot 10}=\frac{3}{4} \cdot 1 \cdot 1=\frac{3}{4} \quad \text { The same answer results. }
$$

now try 9

## OBJECTIVE 3 Convert between improper fractions and mixed numbers.

A mixed number is a single number that represents the sum of a natural number and a proper fraction. The mixed number $2 \frac{3}{4}$ is illustrated in FIGURE 2.

$$
\text { Mixed number } \rightarrow 2 \frac{3}{4}=2+\frac{3}{4}
$$



The mixed number $2 \frac{3}{4}$ is equivalent to the improper fraction $\frac{11}{4}$.

## EXAMPLE 3 Converting an Improper Fraction to a Mixed Number

Write $\frac{59}{8}$ as a mixed number.
Because the fraction bar represents division $\left(\frac{a}{b}=a \div b\right.$, or $\left.b / \bar{a}\right)$, divide the numerator of the improper fraction by the denominator.

$$
\begin{array}{rlrl}
\text { Denominator of fraction } \rightarrow 8 \longdiv { 7 } & \leftarrow \text { Quotient } \\
\frac{56}{3} & \leftarrow \text { Numerator of fraction } & \leftarrow \text { Remainder } & \frac{59}{8}=7 \frac{3}{8} \\
\text { NOW TRY }
\end{array}
$$

## EXAMPLE 4 Converting a Mixed Number to an Improper Fraction

Write $6 \frac{4}{7}$ as an improper fraction.
Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.

$$
7 \cdot 6=42 \quad \text { and } \quad 42+4=46
$$

The denominator of the improper fraction is the same as the denominator in the mixed number, which is 7 here.

$$
6 \frac{4}{7}=\frac{7 \cdot 6+4}{7}=\frac{46}{7}
$$

now try 9

## Multiplying Fractions


$\frac{3}{4}$ of $\frac{1}{2}$ is equivalent to $\frac{3}{4} \cdot \frac{1}{2}$, which equals $\frac{3}{8}$ of the circle.

FIGURE 3

NOW TRY
EXERCISE 5
Find each product, and write it in lowest terms as needed.
(a) $\frac{4}{7} \cdot \frac{5}{8}$
(b) $3 \frac{2}{5} \cdot 6 \frac{2}{3}$

## NOW TRY ANSWERS

5. (a) $\frac{5}{14}$
(b) $\frac{68}{3}$, or $22 \frac{2}{3}$

## OBJECTIVE 4 Multiply and divide fractions.

FIGURE 3 illustrates multiplying fractions.

## Multiplying Fractions

If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions, then $\frac{a}{b} \cdot \frac{c}{\boldsymbol{d}}=\frac{\boldsymbol{a} \cdot \boldsymbol{c}}{\boldsymbol{b} \cdot \boldsymbol{d}}$.
That is, to multiply two fractions, multiply their numerators and then multiply their denominators.

## EXAMPLE 5 Multiplying Fractions

Find each product, and write it in lowest terms as needed.
(a) $\frac{3}{8} \cdot \frac{4}{9}$

$$
\begin{array}{ll}
=\frac{3 \cdot 4}{8 \cdot 9} & \\
=\frac{\text { Multiply numerators. }}{72} & \\
\text { Multiply denominators. } \\
=\frac{12}{6 \cdot 12} & \begin{array}{l}
\text { Multiply. } \\
\text { The greatest common factor } \\
\text { of } 12 \text { and } 72 \text { is } 12 .
\end{array}
\end{array}
$$

$$
\begin{gathered}
\text { Make sure the product } \\
\text { is in lowest terms. }
\end{gathered}=\frac{1}{6} \quad \frac{1 \cdot 12}{6 \cdot 12}=\frac{1}{6} \cdot 1=\frac{1}{6}
$$

Another strategy is to factor and divide out any common factors before multiplying.

$$
\begin{array}{rlr}
\frac{3}{8} \cdot & \frac{4}{9} & \\
& =\frac{3}{2 \cdot 4} \cdot \frac{4}{3 \cdot 3} & \\
& \text { Factor. } \\
& =\frac{1}{2 \cdot 3} & \\
& =\frac{\text { Divide out common factors. Multiply. }}{6} &
\end{array}
$$

(b)

$$
2 \frac{1}{3} \cdot 5 \frac{1}{4}
$$

$$
=\frac{7}{3} \cdot \frac{21}{4} \quad \begin{aligned}
& \text { Write each mixed number } \\
& \text { as an improper fraction. }
\end{aligned}
$$

$$
=\frac{7 \cdot 21}{} \quad \text { Multiply numerators. }
$$

Multiply denominators.

$$
=\frac{7 \cdot 3 \cdot 7}{3 \cdot 4} \quad \text { Factor the numerator. }
$$

Think: $\frac{49}{4}$ means $49 \div 4 .=\frac{49}{4}, \quad$ or $\quad 12 \frac{1}{4} \quad \begin{aligned} & \text { Write in lowest terms } \\ & \text { and as a mixed number. }\end{aligned}$


## Dividing Fractions


$\frac{1}{2} \div 4$ is equivalent to $\frac{1}{2} \cdot \frac{1}{4}$, which equals $\frac{1}{8}$ of the circle.

FIGURE 4

Two numbers are reciprocals of each other if their product is 1.

| $\nabla$ Reciprocals |
| :--- |
| Number Reciprocal <br> $\frac{3}{4}$ $\frac{4}{3}$ <br> $\frac{11}{7}$ $\frac{7}{11}$ <br> $\frac{1}{5}$ 5 , or $\frac{5}{1}$ <br> have a product of 1. For example,  <br> 10, or $\frac{10}{1}$ $\frac{1}{10}$ |

Division is the inverse or opposite of multiplication, and as a result we use reciprocals to divide fractions. FIGURE 4 illustrates dividing fractions.

## Dividing Fractions

If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions, then

$$
\frac{\text { Multiply }}{\frac{\boldsymbol{a}}{\boldsymbol{b}} \div \frac{\boldsymbol{c}}{\boldsymbol{d}}=\frac{\boldsymbol{a}}{\boldsymbol{b}} \cdot \frac{\boldsymbol{d}}{\boldsymbol{c}}} \underset{\substack{\text { by the reciprocal. }}}{\text { by }}
$$

That is, to divide by a fraction, multiply by its reciprocal.

As an example of why this procedure works, we know that

$$
20 \div 10=2 \quad \text { and also that } \quad 20 \cdot \frac{1}{10}=2
$$

The answer to a division problem is a quotient. In $\frac{a}{b} \div \frac{c}{d}$, the first fraction $\frac{a}{b}$ is the dividend, and the second fraction $\frac{c}{d}$ is the divisor.

## EXAMPLE 6 Dividing Fractions

Find each quotient, and write it in lowest terms as needed.
(a) $\frac{3}{4} \div \frac{8}{5}$

$$
\begin{aligned}
& =\frac{3}{4} \cdot \frac{5}{8} \quad \text { Multiply by the reciprocal of the divisor. } \\
& =\frac{3 \cdot 5}{4 \cdot 8} \quad \begin{array}{l}
\text { Multiply numerators. } \\
=\frac{15}{32} \quad \underbrace{\text { Make sure the quotient }}_{\text {Multiply denominators. }} \text { is in lowest terms. }
\end{array}
\end{aligned}
$$

$$
\text { (b) } \begin{array}{rlrl}
\frac{3}{4} & \div \frac{5}{8} & \text { (c) } \begin{aligned}
\frac{5}{8} & \div 10 & & =\frac{5}{8} \cdot \frac{1}{10}
\end{aligned} \begin{array}{l}
\text { Think of } 10 \text { as } \frac{10}{1} \text { here. } \\
\\
\end{array} \begin{aligned}
\text { Multiply by the } \\
\text { reciprocal. }
\end{aligned} \\
& =\frac{3}{4} \cdot \frac{3}{5} & \begin{array}{l}
\text { Multiply by the } \\
\text { reciprocal. }
\end{array} & \\
\text { Multiply and } & & =\frac{5 \cdot 1}{8 \cdot 2 \cdot 5} & \begin{array}{l}
\text { Multiply and } \\
\text { factor. }
\end{array} \\
& =\frac{6}{5}, \quad \text { or } 1 \frac{1}{5} & & =\frac{1}{16}
\end{array}
$$

Find each quotient, and write it in lowest terms as needed.
(a) $\frac{2}{7} \div \frac{8}{9}$
(b) $3 \frac{3}{4} \div 4 \frac{2}{7}$

Adding Fractions


FIGURE 5
(d) $1 \frac{2}{3} \div 4 \frac{1}{2}$

$$
\begin{array}{ll}
=\frac{5}{3} \div \frac{9}{2} & \text { Write each mixed number as an improper fraction. } \\
=\frac{5}{3} \cdot \frac{2}{9} & \text { Multiply by the reciprocal of the divisor. } \\
=\frac{10}{27} & \text { Multiply. The quotient is in lowest terms. }
\end{array}
$$

now try 9

## OBJECTIVE 5 Add and subtract fractions.

The result of adding two numbers is the sum of the numbers. For example, $2+3=5$, so 5 is the sum of 2 and 3 .
figure 5 illustrates adding fractions.

## Adding Fractions

If $\frac{a}{b}$ and $\frac{c}{b}$ are fractions, then $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$.
That is, to find the sum of two fractions having the same denominator, add the numerators and keep the same denominator.

## EXAMPLE 7 Adding Fractions (Same Denominator)

Find each sum, and write it in lowest terms as needed.
(a) $\frac{3}{7}+\frac{2}{7}$
(b) $\frac{2}{10}+\frac{3}{10}$
$=\frac{3+2}{7}$
Add numerators.
Keep the same
$=\frac{2+3}{10}$
Add numerators.
Keep the same
$=\frac{5}{7} \quad$ The sum is in lowest terms.

$$
=\frac{5}{10}
$$

$$
=\frac{1}{2} \quad \text { Write in lowest terms. }
$$

If the fractions to be added do not have the same denominator, we must first rewrite them with a common denominator. For example, to rewrite $\frac{3}{4}$ as an equivalent fraction with denominator 12 , think as follows.

$$
\frac{3}{4}=\frac{?}{12}
$$

We must find the number that can be multiplied by 4 to give 12 . Because $4 \cdot 3=12$, by the second property of 1 we multiply the numerator and the denominator by 3 .

$$
\frac{3}{4}=\frac{3}{4} \cdot 1=\frac{3}{4} \cdot \frac{3}{3}=\frac{3 \cdot 3}{4 \cdot 3}=\frac{9}{12}<\begin{gathered}
\frac{3}{4} \text { is equivalent to } \frac{9}{12} . \\
\text { See FIGURE } 6 .
\end{gathered}
$$

$\frac{3}{4}$ and $\frac{9}{12}$ are equivalent fractions.
FIGURE 6
6. (a) $\frac{9}{28}$
(b) $\frac{7}{8}$
7. $\frac{1}{2}$

NOW TRY ANSWERS

NOTE The process of writing an equivalent fraction is the reverse of writing a fraction in lowest terms.

## Finding the Least Common Denominator (LCD)

To add or subtract fractions with different denominators, find the least common denominator (LCD) as follows.

Step 1 Factor each denominator using prime factors.
Step 2 The LCD is the product of every (different) factor that appears in any of the factored denominators. If a factor is repeated, use the greatest number of repeats as factors of the LCD.

Step 3 Write each fraction with the LCD as the denominator.

## EXAMPLE 8 Adding Fractions (Different Denominators)

Find each sum, and write it in lowest terms as needed.
(a) $\frac{4}{15}+\frac{5}{9}$

Step 1 To find the LCD, factor each denominator using prime factors.

$$
15=5 \cdot 3 \text { and } 9=3 \cdot 3 \quad \text { The different factors are } 3 \text { and } 5
$$

Step 2
$\mathrm{LCD}=5 \cdot 3 \cdot 3=45$
In this example, the LCD needs one factor of 5 and two factors of 3 because the second denominator has two factors of 3 .

Step 3 Write each fraction with 45 as denominator.

$$
\begin{aligned}
& \begin{array}{l}
\text { Make sure the sum } \\
\text { is in lowest terms. }
\end{array} \\
& =\frac{37}{45}
\end{aligned}
$$

(b) $3 \frac{1}{2}+2 \frac{3}{4}$

Method 1

$$
3 \frac{1}{2}+2 \frac{3}{4}
$$

$$
=\frac{7}{2}+\frac{11}{4} \quad \text { Write each mixed number as an improper fraction. }
$$

$$
\begin{array}{rlrl}
\text { Think: } \frac{7}{2} \cdot \frac{2}{2}=\frac{14}{4} & =\frac{14}{4}+\frac{11}{4} & & \text { Find a common denominator. Th } \\
& =\frac{25}{4}, \text { or } 6 \frac{1}{4} & \text { Add. Write as a mixed number. }
\end{array}
$$

$$
\begin{aligned}
& \frac{4}{15}=\frac{4}{15} \cdot \frac{3}{3}=\frac{12}{45} \quad \text { and } \quad \frac{5}{9}=\frac{5}{9} \cdot \frac{5}{5}=\frac{25}{45} \quad \begin{array}{l}
\text { At this stage, the } \\
\text { fractions are not } \\
\text { in lowest terms. }
\end{array} \\
& \frac{4}{15}+\frac{5}{9} \\
& =\frac{12}{45}+\frac{25}{45} \\
& \text { Use the equivalent fractions } \\
& \text { with the common denominator. } \\
& \text { Add numerators. } \\
& \text { Keep the same denominator. }
\end{aligned}
$$

## EXERCISE 8

Find each sum, and write it in lowest terms as needed.
(a) $\frac{5}{12}+\frac{3}{8}$
(b) $3 \frac{1}{4}+5 \frac{5}{8}$

## Subtracting Fractions



FIGURE 7

Method $2 \quad 3 \frac{1}{2}=3 \frac{2}{4}$ Write $3 \frac{1}{2}$ as $3 \frac{2}{4}$. Then add vertically.
$+2 \frac{3}{4}=2 \frac{3}{4}$ Add the whole numbers and the
$+2 \frac{3}{4}=2 \frac{3}{4} \quad$ fractions separately.

$$
5 \frac{5}{4}=5+1 \frac{1}{4}=6 \frac{1}{4}, \quad \text { or } \quad \frac{25}{4} \quad \begin{aligned}
& \text { The same } \\
& \text { answer results. }
\end{aligned}
$$

NOW TRY
The result of subtracting one number from another number is the difference of the numbers. For example, $9-5=4$, so 4 is the difference of 9 and 5 .

FIGURE 7 illustrates subtracting fractions.

## Subtracting Fractions

If $\frac{a}{b}$ and $\frac{c}{b}$ are fractions, then $\quad \frac{a}{b}-\frac{c}{\boldsymbol{b}}=\frac{\boldsymbol{a}-\boldsymbol{c}}{\boldsymbol{b}}$.
That is, to find the difference of two fractions having the same denominator, subtract the numerators and keep the same denominator.

## EXAMPLE 9 Subtracting Fractions

Find each difference, and write it in lowest terms as needed.
(a) $\frac{15}{8}-\frac{3}{8}$

$$
\begin{array}{ll}
=\frac{15-3}{8} & \begin{array}{l}
\text { Subtract numerators. } \\
\text { Keep the same denominator. }
\end{array} \\
=\frac{12}{8} &
\end{array}
$$

(b) $\frac{15}{16}-\frac{4}{9}$

$$
=\frac{15}{16} \cdot \frac{9}{9}-\frac{4}{9} \cdot \frac{16}{16} \quad \begin{aligned}
& \text { Because } 16 \text { and } 9 \text { have no common } \\
& \text { factors except } 1, \text { the LCD is } \\
& 16 \cdot 9=144 .
\end{aligned}
$$

$$
=\frac{135}{144}-\frac{64}{144} \quad \text { Write equivalent fractions. }
$$

$$
=\frac{71}{144} \quad \begin{aligned}
& \text { Subtract numerators. } \\
& \text { Keep the common denominator. }
\end{aligned}
$$

(c) $\frac{7}{18}-\frac{4}{15}$

$$
=\frac{7}{2 \cdot 3 \cdot 3} \cdot \frac{5}{5}-\frac{4}{3 \cdot 5} \cdot \frac{2 \cdot 3}{2 \cdot 3} \quad \begin{aligned}
& 18=2 \cdot 3 \cdot 3 \text { and } 15=3 \cdot 5, \text { so } \\
& \text { the LCD is } 2 \cdot 3 \cdot 3 \cdot 5=90 .
\end{aligned}
$$

$$
=\frac{35}{90}-\frac{24}{90} \quad \text { Write equivalent fractions. }
$$

8. (a) $\frac{19}{24} \quad$ (b) $\frac{71}{8}$, or $8 \frac{7}{8}$

$$
=\frac{11}{90}
$$

